Mathematical connections of students in lesson study and open approach

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Abstract

The purpose of this qualitative research was to describe the mathematical connections of the students based on the framework proposed by Evitts (2004) in lesson study and open approach. The target group included the teachers and 27 fifth grade students of Ban Chonnabot Community School under project of implementing lesson study and open approach. The research instruments included lesson plans, field notes, in-depth interview and observation forms. The data were analyzed, and the conclusions were drawn in a descriptive form. The research found that students’ five kinds of mathematical connections (Modeling, Structural, Representational, Procedure-Concept, and Between Strands of Mathematics) occurred in classroom four phases of the open approach as a teaching approach. In addition, the nature of connections-mindedness is characterized through lesson study derived attributes.

Keywords: lesson study, open approach, mathematical connection

Introduction

For the past 20 years, the lesson study has shown the shift from the teacher - centered instruction to the student - centered one in mathematics and science (Lewis and Tsuchida, 1998; Yoshida, 1999; Takahashi, 2000). Lesson study is the activity used by teachers in Japan to develop theories and collaborative teaching (Isoda, 2010). It is regarded as the routine of the teacher that is continuously carried out, aiming at developing the learners’ learning process along with the teacher professional development. Such practice has been perpetuated in Japan for more than 100 years and become working culture of the Japanese teachers. Stigler and Hiebert (1999) reflected the impact of the Japanese teacher professional development through the lesson study to successfully achieve the goal of the Japanese education reform. The program was carried out with continuity and
aimed at achieving the students’ learning objectives. It also took the classroom context into consideration and emphasized the mutual learning among the teachers involved. One mathematics learning process was the mathematical connections. They were prescribed in mathematics curriculum at the school level (NCTM, 1989; NCTM, 2000; Evitts, 2004; Ministry of Education, 2008).

Mathematical connections were crucial conceptual tools for both the teacher and students (Mousley, 2004). They served as the tool for connecting ideas from other branches of mathematics to assure the learners to conceive mathematics as an approach to understand the world themselves. The approach required the students to play the prime role in conducting mathematical connections in the classroom. It was necessary that the students look at mathematics in a connectional way to see when and how a piece of knowledge is to be used (NCTM, 1989). All these are to help the learners in developing their potential to its peak through the student-centered learning (Ministry of Education, 2008).

Open approach is an innovation originated in Japan aiming at development of high level mathematics thinking of the students. In this approach, every student could learn mathematics in the way relevant to his/her ability along with the self decision in the learning process. They could increase the quality of the process and outcomes concerning mathematics (Nohda, 2000). The approach stresses the fact that the problem solution does not end with a particular answer. The approach to a problem is an important aspect. Classroom condition affects the discussion on the concepts and various ideas of the students (Becker and Shimada, 1997).

Mathematics primary education should be aimed at developing the students’ problem solving ability and promoting their learning how to learn (Okubo, 2007). Teaching is not exclusively for delivering the content but for making the students being able to learn by themselves in the future. The use of “why” question could stimulate the students’ thinking and encourage them to find the reasons (Isoda, 2010). Besides, the students’ approach of knowing was also a success of the attempt to promote the students’ self-learning (Inprasitha, 2010).

In general, a Thai teacher plays the roles of lecturer, informer and demonstrator of the mathematics substances all of which are believed to produce the students’ learning which is regarded the ultimate outcome (Inprasitha, 2007). It does not emphasize the process by which the students could search for knowledge themselves. The teacher has authority and regards himself/herself, the most knowledgeable and accurate person in the field. The students have the roles of accepting the knowledge and adapted themselves to the teacher’s content and method of knowledge transmission. The teachers do not emphasize the process of self-knowledge-acquiring (National Commission on Education, 2000). Most mathematics instructions solely depend on doing the exercises in the textbook. Each of such exercises typically has only one correct answer and provides no opportunity for the students with different ability to take part. In analy-
Inprasitha (2010) stated that lesson study in Thailand has utilized the guidelines proposed in the teacher professional development which suggests 3 phases of the operation. The new space for this study was for applying the lesson study and open approach to the teaching context for constructing the description of the students’ mathematical connections under the framework proposed by Evitts (2004) in the cycle of lesson study. The emphasis on the classroom observation was to acquire understanding of how the students solved the problems by themselves. The collective reflection was the attempt to share the experiences among the concerned sides in facilitating the students’ mathematical connections.

**The Adoption of the Japanese Teacher Professional Development Concept of Lesson Study to be Tried in Thailand**

In Thailand, Inprasitha (2010) has adopted the teacher professional development innovation called ‘lesson study’ to be used in Thailand in 2003 for the first time. He tried it with 15 teaching practicum students who taught mathematics in the school of their practicum. In the same year, Khon Kaen University Center for Research in Mathematics Education (CRME) utilized both the lesson study and the open approach in school. It proposed 3 phases of the instruction as follows:

**Phase 1: The collaboratively designed lesson (Plan).**

At this stage, the researchers and the participating teachers joined in the planning, starting with designing the mathematical problem using open-
ended problem situation. Afterward, the instruction is carried out focusing on open approach which opens for individual student to take part in the mathematical activities collaboratively constructed using open-ended problems.

Phase2: The collaboratively observing the research lesson (Do).
At this stage, after the instructional plan has been actualized in the classroom by one of the team who constructed the plans, there would be classroom observation collaboratively conducted by the research team members and other teachers. The aim was to learn about the students’ thinking process, not the teacher’s performance.

Phase3: The collaboratively reflection on teaching practices (See).
The research team and teachers discussed to reflect on the observation of the classroom teaching in order to improve the instruction and use the revised ideas in the next round of the classroom teaching.

The uniqueness of the lesson study was its gradual and continuous process. It emphasizes the changes of the lessons and requires a certain kind of innovation for such purpose. The adaptive model then integrated the various mathematics teaching methods that are based on open approach to be used for the teacher professional development at the classroom level.

Inprasitha (2010) proposed that the 4 phases of the open approach are as follows:

Phase1: Posing open-ended problems
At this stage, the teacher poses open-ended problems with some media and provided the students time to understand the problem.

Phase2: Students’ self-learning through open-ended problem solving.
It is the stage where the students learn by themselves through the efforts of solving the open-ended problems through various methods. The teacher compiles all the solutions proposed by the students.

Phase3: Whole class discussion and comparison.
At this stage, the students present the solution of the open-ended problems. The teacher recognizes all the solutions proposed by the students and facilitates the connections of all mathematical ideas.

Phase4: Summary through connecting students’ mathematical ideas emergedinthe classroom.
tries to link all the ideas to have the students derive the generalizations, rules and formula in mathematics before having the students to note them down onto the board and in the notebook using their own language.

Research objectives
The research was to describe the mathematical connections of the students based on the framework proposed by Evitts (2004) in lesson study and open approach.
Conceptual framework

Mathematical connections of the students suggested by Evitts’ framework (2004) could be derived through the five following ways:

1. Modeling Connections constitute links between the world of mathematics and the real world (or the daily lives) of students.

2. Structural Connections recognizes the sameness of two mathematical ideas or constructs an important goal for school mathematics and emphasizes mathematical structure.

3. Representational Connections mathematical relationships can be represented in graphical, numerical, symbolic, pictorial and verbal forms.

4. Procedure–concept Connections which is the relationship between mathematics procedural knowledge and conceptual knowledge by which the individual could use to describe or practice to derive principle, formula and other forms of mathematic perception.

5. Connection between Strands of Mathematics which was derived through the close looks at the problem situation and links it to the contents in mathematics.

Figure 1 Conceptual Framework of the students based on the framework proposed by Evitts (2004) in lesson study and open approach.
Methodology

Research Design
This research employed qualitative research with longitudinal study and emphasized the process and meaning utilizing ethnographic study method. The research designed the researcher role to learn and work in the school using the lesson study and open approach during 200-09 academic years as the researcher who would describe the specific characteristics of the target group students. She examined the context and took part in the process of learning unit development along with other teachers, students on their teaching practicum, School coordinator, researchers and observing teachers. The collaboratively design research lessons were taught on every Monday using a Japanese mathematics textbook published by Gakkohtosho Co., Ltd., along with the Thai mathematics textbook. The classroom observation was collaboratively carried out on every Monday and Thursday. The collaborative reflection was done on every Thursday. Regarding the teaching, the researchers had some direct teaching experience on reasoning and mathematics learning of the students from the classroom observation. The study was done on Ban Chonabot Community School, Chonabot District, Khon Kaen Province, a primary school in the Northeast under the Office of Khon Kaen Education Service Area 2 under the Project on Mathematics Teacher. She took role of the teacher in this experiment.

The target group was composed of the 27 fifth grade students, aging between 10–11 years 12 males and 15 females. In the class, students were divided into groups of 4-5 members to engage in the activity without considering their academic achievement. One group consisting 4 students – 2 boys and 2 girls, was chosen on the criterion of their working together well when coping with open-ended problem situation. After being posed to the problem situation, each member of the group would start thinking, using their own thinking mode. They were assertive and were not too concerned about giving out wrong answer. They could describe their way of thinking. According to Piaget’s developmental theories, the children of this age could perform reasoning thinking though they still needed concrete cues. Students of this age begin to have logical thinking and think back and forth. They could see things from more different perspectives and could set up the criteria to classify things.

The teacher was a full time teacher who had taken part in the Project using lesson study and open approach for a 4-year-long period (2006-09) at five grade in mathematics. The team had collaboratively set up the learning plans, engaged in the collaborative observation plans, took part in the collective reflection and regularly engaged in the project activities. Having regularly been in the classroom, all these participants could learn about the students’ thinking process as well as encountered in detail their thinking and reasoning.
**Procedural steps**

The cycle of lesson study is composed of 3 main phases – the collaborative learning unit planning, the collaborative classroom observation and the collaborative reflection.

**Phase1: The collaboratively design research lesson (Plan)**

This phase was participated by the teachers who had jointed the research team, students on teaching practicum, members of the research team from Center for Researcher in Mathematics Education, Faculty of Education, Khon Kaen University and the School’s coordinator. Regarding the learning plans, there were 13 learning modules each of which took one class period on the area of a figure, 4 on parallelogram area and the “How big I am.” activity.

The lesson planning was collaboratively done by the teaching teacher, observant teachers, school coordinator, students on teaching practicum and researchers from the research team discussing the aim of the lesson, creating the open-ended problem situation, with the instructions, order of the tasks, media and equipment, time for each task, the anticipation of the students’ conceptualization, information of the students’ thinking from the previous sessions, material design relevant to the instruction/problem situation and the order for the students to present their ideas, all of which were to facilitate the students’ ideas and mathematics concepts.

**Phase2: The collaboratively observing the research lesson (Do)**

This phase was the implementation of the planned learning modules carried out by the teacher. The observers included the researcher, students on teaching practicum, members of the research team from the Mathematics Education Research Center, Faculty of Education, Khon Kaen University and the School coordinator. The object of the observation was the students’ mathematical connections under the framework proposed by Evitts (2004) covering five kinds and was not to be concerned with the teacher’s teaching competency. The sequence of teaching episodes began with the teacher who was the teaching agent from the team designing the learning modules. The researcher would particularly observe mathematical connections of the target students among mathematics contents through the framework proposed by Evitts (2004) in the atmosphere of the open approach which determined the steps of problem solving at each particular episode. The classroom observers included the students on their teaching practicum at the School, research team members from Center for Researcher in Mathematics Education, Faculty of Education, Khon Kaen University and the School coordinator served as the witness in the teaching episodes. During this, there were some recordings in the form of video, sound and on the observation forms all of which were with copy rights of Center for Researcher in Mathematics Education, Khon Kaen University. The researcher used open approach to guide the observation of each of the sequence of teaching episodes.
Phase 3: The collaboratively reflection on teaching practices (See)

This activity was launched on every Thursday. The reflectors included members of the learning module developing team and the classroom observers. The reflection was based on the recordings of the classroom observation. The results had led to the revision of the learning modules in which the reflective points were to be realized in the classes to come.

Information Collecting Instruments

1. Learning plans on the area of a figure in the area of a parallelogram and the activity on “How big I am.”
2. Forms for recording the students’ behavior, mode of thinking during their problem solving attempt and their explanation of the procedures and method of deriving the answer
3. Interview forms
4. Forms for observing the activity of the team’s collaborative engagement in developing learning plan, classroom observation forms and reflection recording form

Information Collection

The researcher collected the information during the first semester of 2010 academic year from the target group students. The student’s behaviors were recorded without interfering with the thinking process. The teacher taught the class and also carried out the participant observation through the close look and described the occurring phenomena. The writing of the interview questions was conducted formally. The semi-structured interview was conducted immediately with the target group students for 1 hour. During the semi-structured interview, the students were allowed for pauses and time for thinking. There were no interruption, judgment of any response, and suggestion of a particular response encouragement or discouragement answers. Afterward, the researcher interviewed the teacher for 1 hour on the issues of the teacher in posing the open-ended problem, the compilation of the students’ problem-solving methods, class discussion on the problem-solving methods and the organization and the connection of the students’ problem solving. The information collection took 2 hours from 10.00 to 12.00 a.m. The work done by the students also served as the source of information. The researcher took photo of all the work pieces. At the end of the class, the researcher immediately interviewed the students to acquire additional information on their thinking and validated the information.

Results

The mathematics classroom problem solving using open approach covering 4 instructional steps on the area of quadrilateral took 4 teaching periods. The problem situation on “How tall am I” took 1 period in which the students explained the needed lengths to calculate the quadrilateral starting from learning about the transformation.

The learning preparation was needed for the students to find the area of the quadrilateral before the next stages in which the students learned the reshuffle, the cuts to make a parallelogram, the
measurement, and the multiplication of the base and the height.

**Problem Situation “How tall Am I?”**

The goal of the class was to have the students indicate the length needed for finding the area of the quadrilateral and the relationship between the base and the height.

**Instruction**

1. The students measure the length of each side of quadrilaterals a, b and c.
2. The students compare the area of quadrilaterals a, b and c.
3. The students describe different ways of finding the area of quadrilaterals a, b and c as many as they can.
4. The students present their group work in front of the class.

Description of the mathematical connections of the students based on the framework proposed by Evitts (2004) in the cycle of lesson study emphasizing the observation on the students’ learning how to learn is as follows:

1. **Modeling connections**—the recall of familiar things in everyday life and the previous period’s learning to approach mathematics. The students were observed of their attempt in cutting or moving parts to make up a complete unit for finding area of the figure. Students see figure a in complete unit and look at figures b and c things in figure a form. They cut and moved parts of figures b and c to make figure a out of them. Figure a functioned as the thing to start with from which the students were trying to use as the model for manipulating the other two figures.

Student N: We can find the area of figure a by multiplying its length and width.

Student M: (Fixed figure a on the flip paper to be presented.)

Teacher: How could we find the area, Dear? Let’s help one another in finding the area first. Remember? How did we find the area yesterday?

Student M: I think…we…did it by attaching the figures.

Student K: We from figure b from any side and attach it like this. Cut if off and attach the part in.
Student K: (cut and handed the part to Student M to fix on the paper to be presented) How long is the width? How long is the length? Count to 4 at a time. How many times? Let’s figure out.

Teacher: We have only 5 minutes left now.

Student K: We’ll cut from figure c. Let’s get a ruler.

Student N: To find the area of c, after we attached, we count 3 at a time for 6 times. We’ll get 3x6.

2. Structural connections - A structural connections occurred when the students used and analyzed the structure within the formula of a parallelogram which the students took as the complete unit, from which they started to think about the cutting, moving and linking with line to make the complete unit of the problem figure.

Teacher: Besides multiplying like this, do we have any other ways?

Students (whole class): By cutting and fixing on.

Teacher: Come here and point how.

Student K: Right here.

Teacher: Cut and fix in.

What figure do we get, then?

Students (whole class): A rectangular!

3. Representational connections - The mathematical relationship could be shown in various forms such as number, symbol, picture and verbal expression for individual communicative meaning. Representation for finding the area makes use of counting, cutting out and fixing up, using the rectangular area finding formula, etc. Verbal expression carries the meaning for describing how to find the area. In the phase of whole class discussion, verbal expression is needed along with additional media to present the idea to the class or at the stage where the teacher concluded the lesson on method of finding area of a parallelogram.

Teacher: How does the green side has to do with blue side? How are they?

Students(whole class): It stands straight up.

Teacher: Stands up in what way?

Students(whole class): Up vertically.

4. Procedure-concept connections - At this stage, the students took action in procedural steps in obtaining a principle or formula. The students explored the ways for deriving the formula to find the area of the parallelogram. They had found it by cutting and moving parts of the figure
to make complete unit. Through this, the students were connecting to the concept of a unit.

Teacher: Let’s look at these two figures.

I will ask in figures a and b, length of which side we need to use for finding the area?

Student K: We use this one.

Teacher: Which side do we need to use for finding the area?

Student (whole class): The blue line.

Teacher: But what is the blue line?

Student (whole class): The length.

Teacher: What is the green line?

Student (whole class): The width.

Teacher: How could we get the area?

Student (whole class): By multiplying the width with the length.

Teacher: By multiplying the width with the length. How long is the width? How long is the length?

Student (whole class): The length is 6 centimeters long. The width is 5 centimeters long.

6. Connection between Strands of Mathematics - the hiding of the substances in the problem was to create the situation in which the students could discover themselves within the scope of the lesson’s objectives. The hidden substances were geography and number.

Teacher: What kind of figure are b and c?

Students (Whole class): Parallelogram.

Teacher: How could we get the area of figure a?

Students (Whole class): By multiplying width with the length.

Conclusion

The students’ mathematical connections in Evitts’ framework (Evitts, 2004) in lesson study and open approach are as follows:

1. Modeling connections, the students tried to recall what they were familiar with in their everyday life and the previous learning session as the base to step Into the world of mathematics themselves.

2. Structural connections, the students used complete unit to start with and tried to cut, move and link with line to make a complete unit.
3. Representational connections, the students represented the mathematical relationship in terms of form, number, symbol and verbal expression used by the students in communicating ideas.

4. Procedure-concept Connections, the students tried to form the formula to find the area of a parallelogram themselves through the procedural steps that led to the concept of a unit.

5. Connection between Strands of Mathematics, the hiding of substances in the problem situation stimulated the students to find the substances themselves relevant to the objectives of both at the learning module or the class period levels.

**Discussion**

Through open-ended problem situation, the students step into mathematics world of modeling connection through the cognitive construction - to review what they had learned before, and to simplify (Becker and Shimada, 1997). Structural connection was the result of coping with the open-ended problem situation and one could manage it by recalling the episodes and similar problems. It facilitated the student’s thinking (NCTM, 1989). The representational connections demonstrated the students’ thinking in various ways at the self-learning phase. The teacher compiled all of the students’ solutions to the open-ended problems including the class discussion on the solutions are the steps used which comply with the principles proposed by Goldin and Shteingold (2001) who insisted the representation is not comprehensible if the parts are represented separately. Each part belongs to the meaningful whole. The procedure - concept connections to explain one own thinking could lead to the acquisition of principles, formula, perception on mathematics and conceptualization (Coxford, 1995). The connection between strands of mathematics could help the person to develop concepts that link to other strands such as the connections between strands on geometry and number (Geddes and Fortunato, 1993). All of these points to one principle, that is, each student should be equipped with a kind of readiness – the learning how to learn.

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